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Household specialization and competition for promotion*

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Abstract

We study how the presence of promotion competition in the labor market affects household specialization patterns. By embedding a promotion tournament model in a household setting, we show that specialization can emerge as a consequence of competitive work incentives. This specialization outcome, in which only one spouse invests heavily in his or her career, can be welfare superior to a situation in which both spouses invest equally in their careers. The reason is that household specialization reduces the intensity of competition and provides households with consumption smoothing. The specialization result is obtained in a setting where spouses are equally competitive in the labor market and there is no household production. It is also robust to several modifications of the model, such as varying the number of households, two spouses competing for promotion in the same workplace, and the inclusion of household production.

Keywords: contest theory, gender equality, family, household, competition

JEL classification: C72, D13, J16, J71, M51, M52

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1 Introduction

A large literature in labor economics documents gender gaps in labor market outcomes, emphasizing differences in wages, working hours, employment rates, and occupations. Despite a strong convergence process over the last half-century, substantial gender differences in the labor market remain (Olivetti and Petrongolo 2016). While preferences certainly might play a role in explaining this remaining gap, the literature has mostly focused on differences in the opportunities for men and women to succeed in the labor market and the desirability of policies that 'level the playing field'.

In this paper, we show how gender gaps can arise in response to labor market competitiveness even when men and women are equally competitive and have equal opportunities to succeed in the labor market. Our focus is on settings where both spouses in a household face career incentives in the sense that their work effort affects the likelihood of promotions that lead to higher pay and career advancement. Our work is motivated by the prevalence of promotion competition as an incentive system in firms and organizations (Lazear and Rosen, 1981, Waldman, 2013) and the increasing number of dual career households facing such incentives (Costa and Kahn, 2000).[1]

We set up a theoretical model with two identical two-earner families consisting of two identical spouses. Each spouse in the first family competes for promotion against a spouse in the second family. In this stylized but tractable model, we first show that asymmetric equilibria featuring household specialization generally can emerge. We then illustrate that the specialization equilibrium can deliver higher welfare to both households as compared to when spouses in both families adopt the same competitive effort.

The intuition behind the natural emergence of household specialization is twofold. First, the asymmetric equilibrium reduces the intensity of promotion competition within each firm, implying that both households save on effort costs. Second, a situation where only one spouse exerts high effort provides smoothing of family consumption since intermediate events (where one spouse in each household gets promoted) become more likely.[2]

We explore a number of extensions to highlight the robustness of the specialization result. First, we show that specialization results can be obtained even when the number of households competing at the two firms is four instead of two. Second, we show that specialization can also occur when both spouses work at the same firm. In this case, the consumption smoothing motive is replaced by a negative external effect, as one spouse's effort reduces the other spouse's chances of promotion, which favors specialization. Third, we show that specialization survives when households are allowed to maximize a convex combination of individual and household utility. Finally, we find that the specialization result is robust to the inclusion of a household

[1]See also Green and Stokey, 1983, Malcomson, 1984, Baker et al., 1994a,b, Prendergast, 1999, Bognanno, 2001, DeVaro, 2006, and DeVaro et al., 2019.

[2]The consumption insurance channel has previously been highlighted in a non-tournament setting by, e.g., Blundell et al. (2018).

production effort requirement.

In traditional labor supply models, household specialization typically arises as long as one partner has a comparative advantage in market work, typically due to the presence of a household production sector (see Pollak 2013 for a discussion). We show that household specialization can arise due to the presence of promotion competition without modeling household production or imposing asymmetries in spouses' market skills. In particular, this implies that household specialization can arise even if all household production is outsourced to the market (e.g., even in the presence of family-friendly policies that allow workers to combine childbearing with a career).

Our paper is related to Francois (1998), who also considers ex-ante identical men and women, but focuses on explaining gender discrimination as an equilibrium outcome in a setting where men and women select into different jobs (in contrast, in our setting men and women work in identical jobs). We also add a new angle to the literature on labor market investment within families. A prominent strand of this literature discusses the "family investment hypothesis" and how credit constraints (in the case of immigrant workers) can imply labor market behavior where one "primary worker" engages in investment activities and the other partner engages in activities that finance consumption (see, e.g., Baker and Benjamin 1997 and Cobb-Clark and Crossley 2004).

Section 2 presents the model and derives the specialization result. Section 3 discusses the robustness of our results by exploring extensions and modifications of our baseline setting. Section 4 concludes, and the appendix contains analytical results as well as numerical examples.

2 The model

Following Lazear and Rosen (1981), each worker exerts effort to produce output and the worker with the highest output in the tournament is promoted and wins a prize w_P, while the non-promoted worker receives w_{NP}. The output of each worker is equal to $y = e + \epsilon$, where e is individual effort and ϵ is a random component. The ϵ are assumed to be independently and identically distributed with PDF f and CDF F.

The distinguishing feature of our setup is that we embed a Lazear-Rosen tournament in a household setting. More specifically, we consider two families 1 and 2, and two identical firms A and B. In each family, one member works in firm A, while the other member works in firm B. We denote by ik the spouse in family $i \in \{1, 2\}$ who works in firm $k \in \{A, B\}$, and by il the spouse in family i who works in firm $l \in \{A, B\}$, where $k \neq l$. Similarly, we denote by jk the spouse in family $j \in \{1, 2\}$, $j \neq i$, who works in firm $k \in \{A, B\}$, and by jl the spouse in family j who works in firm $l \in \{A, B\}$, $k \neq l$. The tournament prize structure is the same in both firms. The total family income is equal to the sum of the prizes, which, given the two possible prize levels, allows four different configurations of family income given by the pairs (w_P, w_P), (w_P, w_{NP}), (w_{NP}, w_P), and (w_{NP}, w_{NP}).

Adopting the unitary model of household decision making (see, e.g., Becker 1965, Boskin and Sheshinski 1983 and Kleven et al. 2009), the utility of household i is

$$U(b_i, e_{ik}, e_{il}) = u(b_i) - c(e_{ik}) - c(e_{il}), \tag{1}$$

where b_i denotes the total consumption of family i, and $e_{ik} \geq 0$ and $e_{il} \geq 0$ denote the effort expended by the spouses in family i.[3] Furthermore, u is increasing and strictly concave and c is non-decreasing and strictly convex, satisfying $c(0) = 0$ and $c'(0) = 0$.

The assumption that utility is nonlinear in consumption and depends on total household disposable income is key to our analysis as it implies a diminishing return for a family to have both spouses be highly successful in the labor market.

Let $\Delta e_k = e_{ik} - e_{jk}$ be the effort difference in firm k and $\Delta e_l = e_{il} - e_{jl}$ the effort difference in firm l. Furthermore, let $\Delta u_P = u(2w_P) - u(w_P + w_{NP})$ be the consumption utility gain from going from one to two promoted family members, and $\Delta u_{NP} = u(w_P + w_{NP}) - u(2w_{NP})$ be the gain from going from zero to one promoted family member. We also define $\Delta u = \Delta u_P - \Delta u_{NP} = u(2w_P) + u(2w_{NP}) - 2u(w_P + w_{NP})$, which is negative due to the strict concavity of u. In other words, the first promotion in the family is more valuable than the second.

Family i wins the firm-k tournament against family j if $e_{ik} + \epsilon_{ik} > e_{jk} + \epsilon_{jk}$. This event can be rewritten as

$$\epsilon_{jk} < \epsilon_{ik} + e_{ik} - e_{jk} \tag{2}$$

or

$$\epsilon_{jk} - \epsilon_{ik} < e_{ik} - e_{jk}. \tag{3}$$

Since ϵ_{ik} is i.i.d. with PDF f and CDF F, the probability of (2) can be written as

$$\int F\,(x + e_{ik} - e_{jk})\, f\,(x)\, dx. \tag{4}$$

Following Lazear and Rosen (1981), we define G as the CDF of the difference $\epsilon_{jk} - \epsilon_{ik}$. The probability of (3) can be stated as $G\,(e_{ik} - e_{jk})$. Since both (2) and (3) describe the same event, we have $\int F\,(x + e_{ik} - e_{jk})\, f\,(x)\, dx = G\,(e_{ik} - e_{jk})$. In the following, we will use the latter specification because it keeps the presentation simple.[4]

From the perspective of household i, there are four events. The household

1. wins both tournaments (probability $G(\Delta e_k)G(\Delta e_l)$)
2. wins the tournament at firm k but not at firm l (probability $G(\Delta e_k)(1 - G(\Delta e_l))$)
3. wins the tournament at firm l but not at firm k (probability $(1 - G(\Delta e_k))G(\Delta e_l)$)

[3] We recognize that there are other models of family decision making (see the discussion in Chiappori and Lewbel 2015), and in one of our extensions in subsection 3.3 we explore a departure from the unitary model.

[4] Subsection A.5 contains the analysis for more general distribution functions.

4. wins none of the tournaments (probability $(1 - G(\Delta e_k))(1 - G(\Delta e_l)))$

The expected utility of household i from both tournaments is therefore

$$
\begin{aligned}
& G(\Delta e_k)G(\Delta e_l)u(2w_P) + G(\Delta e_k)(1 - G(\Delta e_l))u(w_P + w_{NP}) \\
& + (1 - G(\Delta e_k))G(\Delta e_l)u(w_P + w_{NP}) + (1 - G(\Delta e_k))(1 - G(\Delta e_l))u(2w_{NP}) - c(e_{ik}) - c(e_{il}) \\
& = G(\Delta e_k)G(\Delta e_l)u(2w_P) + (G(\Delta e_k) + G(\Delta e_l) - 2G(\Delta e_k)G(\Delta e_l))\, u(w_P + w_{NP}) \\
& + (1 - G(\Delta e_k) - G(\Delta e_l) + G(\Delta e_k)G(\Delta e_l))\, u(2w_{NP}) - c(e_{ik}) - c(e_{il}) \\
& = G(\Delta e_k)G(\Delta e_l)\Big(\underbrace{u(2w_P) - 2u(w_P + w_{NP}) + u(2w_{NP})}_{\Delta u_P - \Delta u_{NP}}\Big) \\
& + (G(\Delta e_k) + G(\Delta e_l))\left(\underbrace{u(w_P + w_{NP}) - u(2w_{NP})}_{\Delta u_{NP}}\right) + u(2w_{NP}) - c(e_{ik}) - c(e_{il}) \\
& = G(\Delta e_k)G(\Delta e_l)\Delta u_P + [G(\Delta e_k) + G(\Delta e_l) - G(\Delta e_k)G(\Delta e_l)]\Delta u_{NP} + u(2w_{NP}) - c(e_{ik}) - c(e_{il}).
\end{aligned}
\tag{5}
$$

The baseline utility from consumption is $u(2w_{NP})$ and the second term above gives the increase in utility from winning one promotion, while the first term is the additional utility from a second promotion. Household i jointly chooses e_{ik} and e_{il} in order to maximize (5). Family j faces a problem with the same structure, where the probabilities $G(\Delta e_k)$ and $G(\Delta e_l)$ are replaced by $G(-\Delta e_k)$ and $G(-\Delta e_l)$, respectively, and the maximization is performed with respect to e_{jk} and e_{jl} instead.

The outcome of the firm k tournament is determined by e_{ik} and e_{jk} satisfying the first-order conditions

$$
g(\Delta e_k)\Big[G(\Delta e_l)\Delta u_P + \Big(1 - G(\Delta e_l)\Big)\Delta u_{NP}\Big] = c'(e_{ik}) \tag{6}
$$

$$
g(-\Delta e_k)\Big[G(-\Delta e_l)\Delta u_P + \Big(1 - G(-\Delta e_l)\Big)u_{NP}\Big] = c'(e_{jk}). \tag{7}
$$

2.1 The possibility of asymmetric equilibria

The possibility of asymmetric equilibria is not immediately apparent in our fully symmetric model. However, as we will see, asymmetric household specialization equilibria can arise. We focus on such equilibria where the *total* effort is the same in both families.

$$
e_{ik} + e_{il} = e_{jk} + e_{jl} \iff e_{ik} - e_{jk} = -(e_{il} - e_{jl}) \iff \Delta e_k = -\Delta e_l \neq 0. \tag{8}
$$

Conditional on the efforts of family j, if spouse k in household i specializes in market work (in the sense of exerting a high effort in his/her promotion tournament), it must be the case that spouse l in household i specializes in household work or leisure (in the sense of exerting a low effort in his/her promotion tournament).

Assuming g is unimodal and symmetric around zero, using (8), we have $G(\Delta e_l) = 1 - G(-\Delta e_l) = 1 - G(\Delta e_k)$.[5] Together with $c(e) = de^2$ with $d > 0$, equations (6) and (7) can be rewritten as

$$g(\Delta e_k)\Big[\Big(1 - G(\Delta e_k)\Big)\Delta u_P + G(\Delta e_k)\Delta u_{NP}\Big] = 2de_{ik} \tag{9}$$

$$g(\Delta e_k)\Big[G(\Delta e_k)\Delta u_P + \Big(1 - G(\Delta e_k)\Big)\Delta u_{NP}\Big] = 2de_{jk}. \tag{10}$$

Subtracting (10) from (9) and rearranging yields:

$$g(\Delta e_k)\Big[G(\Delta e_k) - \frac{1}{2}\Big] = \frac{d}{-\Delta u}\Delta e_k, \tag{11}$$

where we recall that $\Delta u = \Delta u_P - \Delta u_{NP} < 0$ so the RHS is non-negative for $\Delta e_k \geq 0$. We have the following result:

Proposition 1. *If g is continuous on $\mathbb{R}^+$ and $g(0) > \sqrt{\frac{d}{-\Delta u}}$, then there exists $\Delta e > 0$ such that (11) holds.*

Proof. Based on (11), we define $H(\Delta e) = g(\Delta e)(G(\Delta e) - 1/2) - \alpha \cdot \Delta e$ where $\alpha = \frac{d}{-\Delta u} > 0$. Notice that $H(\Delta e) < 0$ when $\Delta e \to \infty$ (since $G(\Delta e)$ and $g(\Delta e)$ are bounded). What remains to show is that $H(\Delta e) > 0$ for some $\Delta e > 0$. We consider the point $\Delta e = \varepsilon$ where $\varepsilon > 0$ is small such that $g(\varepsilon) > \sqrt{\alpha}$. We get:

$$\begin{aligned} H(\varepsilon) &= g(\varepsilon)\left(\int_{-\infty}^{0} g(t)dt + \int_{0}^{\varepsilon} g(t)dt - 1/2\right) - \alpha\varepsilon = g(\varepsilon)\left(\int_{0}^{\varepsilon} g(t)dt\right) - \alpha\varepsilon \\ &= g(\varepsilon)g(\xi)\varepsilon - \alpha\varepsilon > 0, \end{aligned}$$

for some $\xi \in (0, \varepsilon)$. The last inequality follows because $g(0) > \sqrt{\alpha}$ and g continuous implies $g(x) > \sqrt{\alpha}$ for all $x \in [0, \varepsilon]$. $\square$

2.2 The symmetric equilibrium

A symmetric equilibrium $\hat{e}$ satisfies $\Delta e_k = \Delta e_l = 0$. Insertion into either (6) or (7) yields

$$g(0)\frac{1}{2}\Big[\Delta u_P + \Delta u_{NP}\Big] = c'(\hat{e}). \tag{12}$$

Since this equation has a solution, a symmetric equilibrium candidate generally exists. Note that $\hat{e}$ is the level of effort chosen by both spouses in both families.

[5] A sufficient but not necessary condition for this assumption is that f is unimodal and symmetric around zero.

2.3 Welfare comparison between the two equilibria

Whether the asymmetric or symmetric equilibrium provides higher utility to families depends on how large the *expected consumption utility* and *total effort cost* are in each equilibrium, recalling the definition of household welfare in (1). The expected utility of consumption for family i in a *symmetric equilibrium* is:

$$\frac{1}{4}[u(2w_P) + u(2w_{NP})] + \frac{1}{2}u(w_P + w_{NP}). \tag{13}$$

Due to the strict concavity of u, the first promotion is more valuable than the second, i.e., we have

$$\begin{aligned} & u(w_P + w_{NP}) - u(2w_{NP}) > u(2w_P) - u(w_P + w_{NP}) > 0 \\ \iff & \frac{1}{4}[u(2w_P) + u(2w_{NP})] < \frac{1}{2}u(w_P + w_{NP}), \end{aligned} \tag{14}$$

which shows that the first term in (13) is smaller than the second. We will use this result in the following.

In contrast, when playing the *asymmetric equilibrium*, rearranging (14) and again using $G(\Delta e_l) = 1 - G(-\Delta e_l) = 1 - G(\Delta e_k)$, we get

$$\begin{aligned} & G(\Delta e_k)G(\Delta e_l)u(2w_p) + (G(\Delta e_k)G(\Delta e_l) - G(\Delta e_k) - G(\Delta e_l) + 1)u(2w_{NP}) \\ & + (G(\Delta e_k) + G(\Delta e_l) - 2G(\Delta e_k)G(\Delta e_l))u(w_P + w_{NP}) \\ = & G(\Delta e_k)(1 - G(\Delta e_k)) \cdot [u(2w_P) + u(2w_{NP})] + [(G(\Delta e_k))^2 + (1 - G(\Delta e_k))^2] \cdot u(w_P + w_{NP}). \end{aligned} \tag{15}$$

In this last expression, $G(\Delta e_k)(1-G(\Delta e_k)$ is the probability of winning (resp. losing) both promotions, while $(G(\Delta e_k))^2 + (1 - G(\Delta e_k))^2$ is the probability of getting exactly one promotion for the household. Since $G(0) = \frac{1}{2}$, we have $G(\Delta e_k) > \frac{1}{2}$, implying $G(\Delta e_k)(1-G(\Delta e_k)) < \frac{1}{4}$ and since probabilities add up to one, it follows that $[(G(\Delta e_k))^2 + (1 - G(\Delta e_k))^2] > \frac{1}{2}$. It follows that, compared to (13), the first term in (15) has a smaller probability weight and the second term has a larger weight. The expression (15) will therefore be strictly larger than (13). Therefore, the asymmetric equilibrium always provides "smoothing" of total household consumption because it assigns higher probabilities to outcomes with one promoted spouse per household.

Regarding total effort costs, in Appendix A.3, we show for the uniform distribution that if d is sufficiently large, and u is not too concave, then both efforts in the asymmetric equilibrium are smaller than the symmetric equilibrium effort (provided they coexist), implying that welfare is higher through both the consumption and effort channels.[6]

[6] In general, given asymmetric effort of household j, household i faces asymmetric tournaments in both firms, which typically have lower effort than symmetric tournaments. On the other hand, asymmetric effort implies

2.4 Numerical example

We now provide a numerical example to illustrate that the asymmetric equilibrium can welfare dominate the symmetric equilibrium when they both coexist (see the Appendix for details). Suppose that the noise terms are uniformly distributed on $[-1/2, 1/2]$. Without loss of generality, we assume that family i puts in more effort into the firm-k tournament than family j, namely, $\Delta e_k > 0$.

Table 1 shows the results for $c(e) = e^2$. We fix $u(2w_{NP}) = 2$, $u(w_{NP} + w_P) = 4$ and consider variation in $u(2w_P)$, letting it take on the three values 4.1, 4.5 and 5. Note that for $u(2w_P) = 5$ (third row), equation (11) has the unique solution $\Delta e_k = 0$. In the other two examples, (11) has two solution candidates, i.e., $\Delta e_k = 0$ and $\Delta e_k > 0$, although only one of the candidates is an equilibrium for $u(2w_P) = 4.1$. In the example with $u(2w_P) = 4.5$, we can compare the two equilibria as they exist simultaneously, and we see that the asymmetric equilibrium has lower total effort cost *and* higher expected utility from consumption, showing that the asymmetric equilibrium welfare-dominates the symmetric equilibrium.

Table 1: Numerical example

	symmetric equilibrium			asymmetric equilibrium		
$u(2w_P)$	e_{sym}	total cost	$E[u(b)]$	(e_H, e_L)	total cost	$E[u(b)]$
4.1	–	–	–	(0.516, 0.157)	0.291	3.690
4.5	0.625	0.781	3.625	(0.595, 0.353)	0.478	3.693
5	0.750	1.125	3.750	–	–	–

Note: Illustration of the possibility of having either only an asymmetric equilibrium (first row), only a symmetric equilibrium (third row), or both existing at the same time (second row). The total cost is equal to $2c(e_{sym})$ in the case of a symmetric equilibrium, and equal to $c(e_H) + c(e_L)$ in the case of an asymmetric equilibrium. The second-order conditions have been verified.

2.5 Discussion

We have derived the household specialization result in a game between households whose spouses compete for promotion, where wages (tournament prizes) are exogenously given. Thus, our analysis does not address the question of how firms' wage setting would respond to household behavior. We also do not analyze welfare consequences beyond the household, such as firm profits and social welfare. Thus, we have analyzed a 'slice' of the labor market, focusing on household behavior as a response to given incentive systems in firms.

Compared to the Lazear and Rosen (1981) tournament model, in our setup, one firm's wage setting would affect another firm through the household's decision making. This would lead to strategic interaction between firms at the wage-setting stage. The analysis of a larger game involving firms' wage setting is beyond the scope of our analysis.

higher total effort cost due to convex cost functions.

In a situation like the second row of Table 1, both equilibria coexist for the given wages and the specialization equilibrium is strictly preferred by all households. It is not obvious whether or not it is possible and profitable for the firms to implement a symmetric equilibrium by setting wages. This is especially the case given that the firms' wage decisions interact strategically. The symmetric equilibrium would be attractive to a firm only if the firm's profit net of wages in the symmetric equilibrium is greater than the profit from the specialization equilibrium. This places an upper bound on the wages that the firm would be willing to pay to induce households to abandon specialization. Wage setting as a tool to influence promotion effort would have to overcome the two benefits of specialization: consumption smoothing and effort cost saving, which is, for example, more difficult the more risk-averse households are (the more they value consumption smoothing). Note that this argument ignores household production, which may be another obstacle to moving away from specialization.

3 Extensions

In this section, we consider four extensions/modifications of our model: (i) let the number of households be four instead of two, (ii) let both spouses work at the same workplace, (iii) allow households to maximize a convex combination of individual and household utility, and (iv) introduce a fixed household production effort to be performed by one of the spouses.

3.1 Four households and two firms

We extend the basic setup to four households and two firms, so that the matching is similar to that in the basic model: each household competes for promotion in two different firms, each household facing competition from three other households. The tradeoffs remain similar to those in the base model: For sufficiently concave household utility (which makes the second promotion much less attractive than the first), there are again asymmetric equilibria in which households focus on winning one of the promotion contests, thereby smoothing consumption and reducing total effort costs. Analytical results and a numerical example can be found in Appendix B.1.

3.2 Both spouses at the same workplace

Next, we assume that both spouses in a household compete for promotion in the same firm. This allows us to study the economic trade-offs that arise when household members jointly maximize household utility while competing directly for promotion. We assume that two households work in the firm to allow for competition from outside the household.

In this setup, it is not possible for the household to receive two promotions. Therefore, there is no longer a trade-off between the costs and benefits of one or two promotions as the consumption smoothing motive is absent. Instead, there is a negative intra-household externality that is not present in the base model: An increase in one spouse's effort makes it less likely that the other spouse will be promoted. It turns out that this effect, combined with competition from the other household, again provides the conditions for the existence of an asymmetric equilibrium. Appendix B.2 contains the analysis and a numerical example.

3.3 Maximizing a combination of individual and household utility

We have used a unitary household model in which both spouses maximize a joint utility function, implying joint risk aversion to consumption opportunities for the household as a whole. In the following, we discuss the robustness of our results when we depart from the unitary model by including an element of individual utility maximization.

To investigate the robustness of our results, and to preserve the base model as a special case, we consider a model extension in which each spouse maximizes a weighted average (convex combination) of the unitary household payoff of the base game (weight α) and an individual payoff $\tilde{u}$ (weight $1-\alpha$) that depends on whether the spouse is promoted, in which case the utility is $\tilde{u}(w_P)$, or not promoted, in which case the utility is $\tilde{u}(w_{NP})$. Appendix B.3 contains the analysis and a numerical example which shows that by reducing the weight of joint household utility maximization from 100% (base model) to 70%, we still obtain household specialization.

3.4 Introducing a household production effort requirement

Next, we assume that there is a fixed amount of household production effort, $\bar{e}_{HP} > 0$, that must be allocated to one of the spouses in each household (e.g., late pregnancy and childbearing effort). This implies that the cost function of that spouse changes from $c(e)$ to $c(e+\bar{e}_{HP})$, while the costs of the other spouse and the contests at the two firms remain unchanged. We abstract from the utility that this effort provides (e.g., in the form of household public goods).

Appendix B.4 demonstrates that we obtain a specialization equilibrium $\tilde{e}_H$ and $\tilde{e}_L + \bar{e}_{HP}$ in the modified game such that, in each household, one of the spouses again focuses on getting promoted by exerting a high effort ($\tilde{e}_H$), while the other spouse contributes the household production effort and exerts a smaller promotion effort ($\tilde{e}_L < \tilde{e}_H$). The example shows that both spouses reduce their promotion effort compared to the initial equilibrium.

The spouse with the lower effort reduces the promotion effort due to the convexity of the effort cost function, while the spouse with the higher effort reduces the promotion effort due to an indirect effect: For this spouse, the change comes from less intense competition in the firm, since this spouse now faces a rival that is engaged in household production and therefore exerts less promotional effort. Thus, in this setup, home production can be seen as a commitment device for lower promotion effort, which in turn has the effect of making the rat race less intense for all parties involved.

It is worth noting that the spouse with the lower promotion effort not only contributes to the household production effort, but also bears the economic burden of household production. This is illustrated by our numerical example in Appendix B.4, in which the promotion effort of the spouse engaged in household production experiences an 86% reduction, while the other spouse experiences a 15% reduction, compared to the game without household production. Thus, the introduction of household production, as we have modeled it, leads to a large reduction in the career orientation of one of the spouses. While we have only considered a simple version of household production, it seems intuitive that household specialization goes hand in hand with asymmetric sharing of household production.

4 Concluding remarks

Promotion tournaments are a common incentive structure in competitive labor markets. In a fully symmetric model with identical spouses, we show that household specialization can arise when spouses participate in such tournaments. The specialization equilibrium can be welfare-dominant over the symmetric equilibrium as it allows both households to save on effort costs and provides consumption smoothing benefits.

Our setting is stylized, but provides the interesting result that household specialization can arise for dual-career couples who are equally competitive and face the same labor market circumstances. It shows that household specialization can reflect an efficient response to firms' incentive systems. Notably, the specialization result is obtained in the absence of household production, i.e., even when an important reason for household specialization is absent.

We have examined the robustness of the specialization result in a number of extensions, increasing the number of competing households, considering spouses working at the same workplace, changing the objective function of the household, and introducing a fixed household production effort to be performed by one of the spouses.

In our paper, the gender identity of each spouse is not specified. However, given empirically documented gendered patterns of household specialization, our analysis suggests that substantial gender differences in labor market outcomes may persist even when men and women have equal opportunities to succeed in the labor market. It seems intuitive that, especially in occupations that reward long and particular hours (Goldin, 2014), specialization may be expected and even necessary given the nature of competition.

In conclusion, there are aspects not included in our analysis that are likely to work against specialization. One such aspect is household public goods and preferences for shared leisure time, which may favor a more equal distribution of work effort within the household. Another is the pursuit of a career as a form of insurance against marital failure. Finally, in reality there may be a lower bound on the effort that a career-oriented spouse is willing to accept, limiting the degree of specialization in households where both spouses wish to pursue careers.

References

Baker, G., Gibbs, M., and Holmström, B. (1994a). The internal economics of the firm: Evidence from personnel data. *Quarterly Journal of Economics*, 109(4):881–919.

Baker, G., Gibbs, M., and Holmström, B. (1994b). The wage policy of a firm. *The Quarterly Journal of Economics*, 109(4):921–955.

Baker, M. and Benjamin, D. (1997). The role of the family in immigrants' labor-market activity: An evaluation of alternative explanations. *American Economic Review*, pages 705–727.

Becker, G. S. (1965). A theory of the allocation of time. *The Economic Journal*, 75(299):493–517.

Blundell, R., Pistaferri, L., and Saporta-Eksten, I. (2018). Children, time allocation, and consumption insurance. *Journal of Political Economy*, 126(S1):S73–S115.

Bognanno, M. L. (2001). Corporate tournaments. *Journal of Labor Economics*, 19(2):290–315.

Boskin, M. J. and Sheshinski, E. (1983). Optimal tax treatment of the family: Married couples. *Journal of Public Economics*, 20(3):281–297.

Chiappori, P. and Lewbel, A. (2015). Gary Becker's a theory of the allocation of time. *Economic Journal*, 125(583):410–442.

Cobb-Clark, D. and Crossley, T. F. (2004). Revisiting the family investment hypothesis. *Labour Economics*, 11(3):373–393.

Costa, D. L. and Kahn, M. E. (2000). Power couples: Changes in the locational choice of the college educated, 1940–1990. *The Quarterly Journal of Economics*, 115(4):1287–1315.

DeVaro, J. (2006). Strategic promotion tournaments and worker performance. *Strategic Management Journal*, 27(8):721–740.

DeVaro, J., Kauhanen, A., and Valmari, N. (2019). Internal and external hiring. *ILR Review*, 72(4):981–1008.

Francois, P. (1998). Gender discrimination without gender difference: Theory and policy responses. *Journal of Public Economics*, 68(1):1–32.

Goldin, C. (2014). A grand gender convergence: Its last chapter. *American Economic Review*, 104(4):1091–1119.

Green, J. R. and Stokey, N. L. (1983). A comparison of tournaments and contracts. *Journal of Political Economy*, 91(3):349–364.

Kleven, H. J., Kreiner, C. T., and Saez, E. (2009). The optimal income taxation of couples. *Econometrica*, 77(2):537–560.

Lazear, E. P. and Rosen, S. (1981). Rank-order tournaments as optimum labor contracts. *Journal of Political Economy*, 89(5):841–864.

Malcomson, J. M. (1984). Work incentives, hierarchy, and internal labor markets. *Journal of Political Economy*, 92(3):486–507.

Olivetti, C. and Petrongolo, B. (2016). The evolution of gender gaps in industrialized countries. *Annual Review of Economics*, 8(1):405–434.

Pollak, R. A. (2013). Allocating household time: When does efficiency imply specialization? *National Bureau of Economic Research*, (19178).

Prendergast, C. (1999). The provision of incentives in firms. *Journal of Economic Literature*, 37(1):7–63.

Waldman, M. (2013). Classic promotion tournaments versus market-based tournaments. *International Journal of Industrial Organization*, 31(3):198–210.

Appendix

A Additional results and derivations

A.1 Details on the triangular distribution

In the numerical example in subsection 2.4, we assume that f is the PDF of the Uniform distribution on $[-1/2, 1/2]$. This implies that the differences of noise terms follow a Triangular distribution with PDF g and CDF G given by

$$g(\Delta e) = \begin{cases} \Delta e + 1, & \text{for} \quad -1 \leq \Delta e < 0 \\ 1, & \text{for} \quad \Delta e = 0 \\ 1 - \Delta e, & \text{for} \quad 0 < \Delta e \leq 1 \\ 0, & \text{for} \quad \Delta e \notin [-1, 1] \end{cases}$$

$$G(\Delta e) = \begin{cases} 0, & \text{for} \quad \Delta e < 0 \\ \frac{(\Delta e + 1)^2}{2}, & \text{for} \quad -1 \leq \Delta e < 0 \\ \frac{1}{2}, & \text{for} \quad \Delta e = 0 \\ 1 - \frac{(1-\Delta e)^2}{2}, & \text{for} \quad 0 < \Delta e < 1 \\ 1, & \text{for} \quad \Delta e \geq 1. \end{cases}$$

A.2 Deriving efforts in the numerical uniform distribution example

Given the assumptions made in the beginning of Section 2.4, the symmetric equilibrium effort $\hat{e}$ is given by the solution of (12):

$$\hat{e} = \frac{1}{4d}(\Delta u_P + \Delta u_{NP}). \tag{A.1}$$

Supposing that the asymmetric candidate satisfies $\Delta e_k \in [0, 1]$ we can re-write (11) as:

$$(1 - \Delta e_k)\Big[\frac{(1-\Delta e_k)^2}{2} - \frac{1}{2}\Big] = \frac{d}{\Delta u}\Delta e_k,$$

which has the obvious solution $\Delta e_k = 0$ (the symmetric equilibrium candidate). The unique solution satisfying $\Delta e_k \in (0, 1]$ is:

$$\Delta e_k^{\text{asym}} = \frac{3}{2} - \sqrt{\frac{1}{4} - \frac{2d}{\Delta u}}, \tag{A.2}$$

provided $-\Delta u > d$. Denote the high effort in the asymmetric equilibrium by e_H and the low effort by e_L, such that $\Delta e_k^{\text{asym}} = e_H - e_L > 0$. Plugging Δe_k^{asym} into (9) allows us to solve for

$e_H = e_{ik}^{\text{asym}}$ (which, by construction, is identical to e_{jl}^{asym}):

$$e_H = e_{ik}^{\text{asym}} = \frac{\Delta u}{4d} \left(1 - \Delta e_k^{\text{asym}}\right)^3 + \frac{\Delta u_{NP}}{2d} \left(1 - \Delta e_k^{\text{asym}}\right). \tag{A.3}$$

Here we notice that the first term is negative and the second term is positive given that $(1 - \Delta e_k^{\text{asym}}) \in [0, 1]$ and $\Delta u < 0$. The low effort e_L is found by solving $\Delta e_k^{\text{asym}} = e_H - e_L$.

A.3 Welfare comparison in the case of a uniform distribution

Here we provide some additional results for the case in which F follows a Uniform distribution.

Proposition 2. *Impose the distributional assumptions of Section 2.4. Then, if d is sufficiently large, and u is not too concave:*

(i) $e_{ik}^{asym} = e_{jl}^{asym} < \hat{e}$ and $e_{jk}^{asym} = e_{il}^{asym} < \hat{e}$.

(ii) The asymmetric equilibrium provides higher welfare to both families as compared to the symmetric equilibrium in the cases where both equlibria exist.

Proof. We begin with Part (i). Notice that when d approaches its upper bound, $-\Delta u$, we have that $\Delta e_{ik}^{\text{asym}} \to 0$ (see equation A.2) and $e_{ik}^{\text{asym}} \to \hat{e}$ (recall (A.1)). Now consider a value of d slightly lower than $-\Delta u$, namely, $d = -\Delta u - \delta$ where $\delta > 0$ is small. This implies that $\Delta e_{ik}^{\text{asym}} = \varepsilon$ where $\varepsilon > 0$ also is small and is a function of δ. We then have from (A.3) that:

$$e_{ik}^{\text{asym}} = \frac{\Delta u}{4d} (1 - \varepsilon)^3 + \frac{\Delta u_{NP}}{2d} (1 - \varepsilon) \approx \frac{\Delta u}{4d}(1 - 3\varepsilon) + \frac{\Delta u_{NP}}{2d} (1 - \varepsilon), \tag{A.4}$$

where the approximation follows by neglecting terms of order ε^2 and ε^3. Subtracting (A.1) from the approximated effort in (A.4) and re-arranging yields:

$$e_{ik}^{\text{asym}} - \hat{e} \approx -\frac{\varepsilon}{4d} (3\Delta u_P - \Delta u_{NP}) \tag{A.5}$$

which is negative provided $3\Delta u_P > \Delta u_{NP}$. This condition amounts to requiring u not to be too concave. To see this, notice that:

$$\begin{aligned} 3\Delta u_P - \Delta u_{NP} &= 3\Big[u(2w_P) - u(w_P + w_{NP})\Big] - \Big[u(w_P + w_{NP}) - u(2w_{NP})\Big] \\ &= 3u(2w_P) + u(2w_{NP}) - 4u(w_P + w_{NP}), \end{aligned}$$

which will be strictly positive for any u that is not too concave. Finally, notice that by virtue of our assumption (without loss of generality) that family i puts in more effort into the firm-k tournament than family j, we also have that $e_{jk}^{\text{asym}} = e_{il}^{\text{asym}} < \hat{e}$. Thus, in the asymmetric equilibrium, both spouses in both families save on effort costs compared to the symmetric equilibrium.

Regarding Part (ii), this follows from Part (i) combined with the fact that the expected utility from consumption is strictly higher in the asymmetric equilibrium (see Section 2.3) due to the strict concavity of u. □

A.4 The reflected exponential distribution

In Sections B.1–B.4, for computational tractability, we use the Reflected Exponential distribution to compute numerical examples.

The Reflected Exponential distribution has support $(-\infty, 0]$ and scale parameter λ. The PDF f and CDF F are given by

$$f(x) = \begin{cases} \lambda e^{\lambda x} & x \leq 0 \\ 0 & x > 0, \end{cases} \qquad F(x) = \begin{cases} e^{\lambda x} & x \leq 0 \\ 1 & x > 0 \end{cases} \tag{A.6}$$

A.5 Analysis of the base game for general distributions

Proposition 1 in the main text was derived using the notation G (as in Lazear and Rosen 1981). In the present section, we present the analysis in terms of distributions F.

In the base game, the expected payoff of household 1 is

$$\begin{aligned}
&\int F(x+e_{1A}-e_{2A})f(x)dx \cdot \int F(x+e_{1B}-e_{2B})f(x)dx \cdot u(2w_P) \\
&+ \int F(x+e_{1A}-e_{2A})f(x)dx \cdot \left(1 - \int F(x+e_{1B}-e_{2B})f(x)dx\right) \cdot u(w_P+w_{NP}) \\
&+ \left(1 - \int F(x+e_{1A}-e_{2A})f(x)dx\right) \cdot \int F(x+e_{1B}-e_{2B})f(x)dx \cdot u(w_P+w_{NP}) \\
&+ \left(1 - \int F(x+e_{1A}-e_{2A})f(x)dx\right) \cdot \left(1 - \int F(x+e_{1B}-e_{2B})f(x)dx\right) \cdot u(2w_{NP}) \\
&- c(e_{1A}) - c(e_{1B}).
\end{aligned} \tag{A.7}$$

The first-order conditions with respect to e_{1A} and e_{1B} are

$$\begin{aligned}
&\int f(x+e_{1A}-e_{2A})f(x)dx \cdot \int F(x+e_{1B}-e_{2B})f(x)dx \cdot u(2w_P) \\
&+ \int f(x+e_{1A}-e_{2A})f(x)dx \cdot \left(1 - \int F(x+e_{1B}-e_{2B})f(x)dx\right) \cdot u(w_P+w_{NP}) \\
&- \int f(x+e_{1A}-e_{2A})f(x)dx \cdot \int F(x+e_{1B}-e_{2B})f(x)dx \cdot u(w_P+w_{NP}) \\
&- \int f(x+e_{1A}-e_{2A})f(x)dx \cdot \left(1 - \int F(x+e_{1B}-e_{2B})f(x)dx\right) \cdot u(2w_{NP}) = c'(e_{1A})
\end{aligned} \tag{A.8}$$

and

$$\begin{aligned}
&\int F(x+e_{1A}-e_{2A})f(x)dx \cdot \int f(x+e_{1B}-e_{2B})f(x)dx \cdot u(2w_P) \\
&- \int F(x+e_{1A}-e_{2A})f(x)dx \cdot \int f(x+e_{1B}-e_{2B})f(x)dx \cdot u(w_P+w_{NP}) \\
&+ \left(1-\int F(x+e_{1A}-e_{2A})f(x)dx\right) \cdot \int f(x+e_{1B}-e_{2B})f(x)dx \cdot u(w_P+w_{NP}) \\
&- \left(1-\int F(x+e_{1A}-e_{2A})f(x)dx\right) \cdot \int f(x+e_{1B}-e_{2B})f(x)dx \cdot u(2w_{NP}) = c'(e_{1B}).
\end{aligned} \tag{A.9}$$

The two conditions can be simplified to

$$\begin{aligned}
&\int f(x+e_{1A}-e_{2A})f(x)dx \cdot \int F(x+e_{1B}-e_{2B})f(x)dx \cdot \Delta u_P \\
&+ \int f(x+e_{1A}-e_{2A})f(x)dx \cdot \left(1-\int F(x+e_{1B}-e_{2B})f(x)dx\right) \cdot \Delta u_{NP} = c'(e_{1A})
\end{aligned} \tag{A.10}$$

and

$$\begin{aligned}
&\int F(x+e_{1A}-e_{2A})f(x)dx \cdot \int f(x+e_{1B}-e_{2B})f(x)dx \cdot \Delta u_P \\
&+ \left(1-\int F(x+e_{1A}-e_{2A})f(x)dx\right) \cdot \int f(x+e_{1B}-e_{2B})f(x)dx \cdot \Delta u_{NP} = c'(e_{1B}).
\end{aligned} \tag{A.11}$$

Note that, using the notation $G(s) = \int F(x+s)f(x)dx$, these conditions can be written as (similar to the presentation in the main text)

$$\begin{aligned}
g(e_{1A}-e_{2A}) \cdot [G(e_{1B}-e_{2B}) \cdot \Delta u_P + (1-G(e_{1B}-e_{2B})) \cdot \Delta u_{NP}] = c'(e_{1A}) \\
g(e_{1B}-e_{2B}) \cdot [G(e_{1A}-e_{2A}) \cdot \Delta u_P + (1-G(e_{1A}-e_{2A})) \cdot \Delta u_{NP}] = c'(e_{1B}).
\end{aligned} \tag{A.12}$$

Now consider the asymmetric 'specialization' candidate $((e_{1A}, e_{1B}), (e_{2A}, e_{2B})) = ((e_H, e_L), (e_L, e_H))$. Denoting $\Delta := e_H - e_L$, such that $e_{1A} - e_{2A} = \Delta$ and $e_{1B} - e_{2B} = -\Delta$, and inserting into (A.10) and (A.11), the two first-order conditions can be written as

$$\begin{aligned}
\int f(x+\Delta)f(x)dx \left[\int F(x-\Delta)f(x)dx \cdot \Delta u_P + \left(1-\int F(x-\Delta)f(x)dx\right)\Delta u_{NP}\right] = c'(e_H) \\
\int f(x-\Delta)f(x)dx \left[\int F(x+\Delta)f(x)dx \cdot \Delta u_P + \left(1-\int F(x+\Delta)f(x)dx\right)\Delta u_{NP}\right] = c'(e_L).
\end{aligned} \tag{A.13}$$

Recalling $c'(e) = 2de$, subtracting the second from the first line above, we get

$$\begin{aligned}
&\int f(x+\Delta)f(x)dx \left[\int F(x-\Delta)f(x)dx \cdot \Delta u_P + \left(1 - \int F(x-\Delta)f(x)dx\right) \Delta u_{NP}\right] \\
&- \int f(x-\Delta)f(x)dx \left[\int F(x+\Delta)f(x)dx \cdot \Delta u_P + \left(1 - \int F(x+\Delta)f(x)dx\right) \Delta u_{NP}\right] \\
&= 2d\Delta,
\end{aligned} \tag{A.14}$$

i.e., a single equation that depends on Δ only. This equation can be used to numerically identify asymmetric equilibrium candidates. These Δ-candidates can then be reinserted into (A.13) to get candidates for e_H and e_L.

For example, assuming the Reflected Exponential distribution (A.6) with scale parameter $\lambda = 2$ as well as $u(2w_{NP}) = 1$ and cost function $c(e) = e^2$, a symmetric equilibrium (but no asymmetric equ.) is obtained for $(u(w_P + w_{NP}), u(2w_P)) = (2, 2.1)$ and an asymmetric 'specialization' equilibrium (but no symmetric equ.) for $(u(w_P + w_{NP}), u(2w_P)) = (3, 3.1)$.

B Extension derivations

B.1 Four households and two firms

Assume that four households compete for promotion in two firms, so that each household is 'split' between two firms, i.e., the two members of each household work in different firms. This is the natural extension of the main two-by-two model, where household members also compete at different firms. Thus, there are four workers competing in each firm, and only one of them is promoted. Again, the promoted worker receives w_P, while the three non-promoted workers each receive w_{NP}.

Denote the set of households by $\mathcal{N} = \{1, 2, 3, 4\}$ and the firms by A and B. Household $i \in \mathcal{N}$ wins the firm-k tournament if i outperforms the other three households,

$$e_{ik} + \epsilon_{ik} > e_{jk} + \epsilon_{jk} \quad \forall j \in \mathcal{N} \setminus \{i\},$$

which we write as $\Delta e_{ijk} + \epsilon_{ik} > \epsilon_{jk}$ for all $j \in \mathcal{N} \setminus \{i\}$, where $\Delta e_{ijk} := e_{ik} - e_{jk}$. The winning (promotion) probability for household i at firm k can thus be stated as

$$\int \prod_{j \in \mathcal{N} \setminus \{i\}} F\left(x + \Delta e_{ijk}\right) f\left(x\right) dx.$$

The expected payoff for family i becomes

$$
\begin{aligned}
&\left(\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijA}\right) f\left(x\right) dx\right) \left(\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijB}\right) f\left(x\right) dx\right) u\left(2w_P\right) \\
&+\left(\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijA}\right) f\left(x\right) dx\right) \left(1-\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijB}\right) f\left(x\right) dx\right) u\left(w_P+w_{NP}\right) \\
&+\left(1-\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijA}\right) f\left(x\right) dx\right) \left(\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijB}\right) f\left(x\right) dx\right) u\left(w_P+w_{NP}\right) \\
&+\left(1-\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijA}\right) f\left(x\right) dx\right) \left(1-\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijB}\right) f\left(x\right) dx\right) u\left(2w_{NP}\right) \\
&-c\left(e_{iA}\right)-c\left(e_{iB}\right).
\end{aligned}
\tag{B.1}
$$

Using Δu_P and Δu_{NP} (as introduced in the main text), this can be simplified to

$$
\begin{aligned}
&\left(\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijA}\right) f\left(x\right) dx\right) \left(\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijB}\right) f\left(x\right) dx\right) \left(\Delta u_P-\Delta u_{NP}\right) \\
&+\left(\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijA}\right) f\left(x\right) dx+\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijB}\right) f\left(x\right) dx\right) \Delta u_{NP} \\
&+u\left(2w_{NP}\right)-c\left(e_{iA}\right)-c\left(e_{iB}\right).
\end{aligned}
\tag{B.2}
$$

The first-order condition for household i regarding the contest at firm k can be stated as

$$
\begin{aligned}
&\frac{\partial\left(\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijk}\right) f\left(x\right) dx\right)}{\partial e_{ik}} \left(\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijl}\right) f\left(x\right) dx\right) \left(\Delta u_P-\Delta u_{NP}\right) \\
&+\frac{\partial\left(\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijk}\right) f\left(x\right) dx\right)}{\partial e_{ik}} \Delta u_{NP}=c'\left(e_{ik}\right),
\end{aligned}
\tag{B.3}
$$

which can be written as

$$
\begin{aligned}
&\frac{\partial\left(\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijk}\right) f\left(x\right) dx\right)}{\partial e_{ik}} \cdot \\
&\quad\left(\left(\int \prod_{j\in\mathcal{N}\setminus\{i\}} F\left(x+\Delta e_{ijl}\right) f\left(x\right) dx\right) \left(\Delta u_P-\Delta u_{NP}\right)+\Delta u_{NP}\right)=c'\left(e_{ik}\right).
\end{aligned}
\tag{B.4}
$$

Writing the two first-order conditions explicitly for household 1 (which chooses efforts e_{1A}

and e_{1B}), we have

$$\begin{aligned}
&\Big(\int f(x+e_{1A}-e_{2A})F(x+e_{1A}-e_{3A})F(x+e_{1A}-e_{4A})f(x)dx \\
&\quad \cdot \int F(x+e_{1A}-e_{2A})f(x+e_{1A}-e_{3A})F(x+e_{1A}-e_{4A})f(x)dx \\
&\quad \cdot \int F(x+e_{1A}-e_{2A})F(x+e_{1A}-e_{3A})f(x+e_{1A}-e_{4A})f(x)dx\Big) \\
&\cdot \Big(\int F(x+e_{1B}-e_{2B})F(x+e_{1B}-e_{3B})F(x+e_{1B}-e_{4B})f(x)dx\,(\Delta u_P-\Delta u_{NP})+\Delta u_{NP}\Big) \\
&= c'\left(e_{1A}\right)
\end{aligned} \tag{B.5}$$

and

$$\begin{aligned}
&\Big(\int f(x+e_{1B}-e_{2B})F(x+e_{1B}-e_{3B})F(x+e_{1B}-e_{4B})f(x)dx \\
&\quad \cdot \int F(x+e_{1B}-e_{2B})f(x+e_{1B}-e_{3B})F(x+e_{1B}-e_{4B})f(x)dx \\
&\quad \cdot \int F(x+e_{1B}-e_{2B})F(x+e_{1B}-e_{3B})f(x+e_{1B}-e_{4B})f(x)dx\Big) \\
&\cdot \Big(\int F(x+e_{1A}-e_{2A})F(x+e_{1A}-e_{3A})F(x+e_{1A}-e_{4A})f(x)dx\,(\Delta u_P-\Delta u_{NP})+\Delta u_{NP}\Big) \\
&= c'\left(e_{1B}\right).
\end{aligned} \tag{B.6}$$

Again, we are interested in an asymmetric equilibrium candidate in which each household specializes, i.e., chooses effort profile (e_H, e_L), while in each firm there are now two high and two low efforts, competing for promotion. An arbitrary candidate effort profile of this kind is

$$((e_{1A},e_{1B}),(e_{2A},e_{2B}),(e_{3A},e_{3B}),(e_{4A},e_{4B})) = ((e_H,e_L),(e_H,e_L),(e_L,e_H),(e_L,e_H)), \tag{B.7}$$

i.e., each household chooses efforts e_H and e_L, while at each firm, we have efforts e_L, e_L, e_H, e_H.

Inserting candidate (B.7) into (B.5) and (B.6) and denoting $\Delta := e_H - e_L$ we get (where, e.g., $F(x+\Delta)^2$ means $F(x+\Delta)\cdot F(x+\Delta)$)

$$\begin{aligned}
&\Big(\int F(x+\Delta)^2 f(x)^2 dx + 2\int F(x+\Delta)f(x+\Delta)F(x)f(x)dx\Big) \\
&\quad \cdot \Big(\int F(x-\Delta)^2 F(x)f(x)dx\,(\Delta u_P-\Delta u_{NP})+\Delta u_{NP}\Big) = c'\left(e_H\right).
\end{aligned} \tag{B.8}$$

and

$$\left(\int F(x-\Delta)^2 f(x)^2 dx + 2\int F(x-\Delta) f(x-\Delta) F(x) f(x) dx\right) \\ \cdot \left(\int F(x+\Delta)^2 F(x) f(x) dx \left(\Delta u_P - \Delta u_{NP}\right) + \Delta u_{NP}\right) = c'\left(e_L\right). \tag{B.9}$$

For each household, we would obtain a similar pair of first-order conditions.

As $c'(e) = 2de$, subtracting (B.9) from (B.8), we obtain $2d\Delta$ on the RHS:

$$\begin{aligned} &\left(\int F(x+\Delta)^2 f(x)^2 dx + 2\int F(x+\Delta) f(x+\Delta) F(x) f(x) dx\right) \\ &\quad\cdot \left(\int F(x-\Delta)^2 F(x) f(x) dx \left(\Delta u_P - \Delta u_{NP}\right) + \Delta u_{NP}\right) \\ &- \left(\int F(x-\Delta)^2 f(x)^2 dx + 2\int F(x-\Delta) f(x-\Delta) F(x) f(x) dx\right) \\ &\quad\cdot \left(\int F(x+\Delta)^2 F(x) f(x) dx \left(\Delta u_P - \Delta u_{NP}\right) + \Delta u_{NP}\right) = 2d\Delta. \end{aligned} \tag{B.10}$$

Equation (B.10) only depends on Δ, so it allows us to use numerical methods to find candidates for asymmetric ($\Delta > 0$) equilibria of the form described in (B.7), where $e_H > e_L$. Reinserting Δ-candidates into (B.8) and (B.9) delivers candidates for e_H and e_L that can be verified numerically by replacing the, respectively, other households' efforts in (B.2) by e_H and e_L (according to (B.7)) and simultaneously maximizing over e_{iA} and e_{iB} to confirm the candidates.

A *symmetric* equilibrium candidate (in which all eight players choose the same effort) can be found directly from equations (B.8) and (B.9) by inserting $\Delta = 0$.

For example, assuming the Reflected Exponential distribution (A.6) with scale parame-ter $\lambda = 2$ as well as $u(2w_{NP}) = 1$ and cost function $c(e) = e^2$, a symmetric equilib-rium (but no asymmetric equilibrium) is obtained for $(u(w_P + w_{NP}),\ u(2w_P)) = (1.2,\ 1.21)$ and an asymmetric 'specialization' equilibrium (but no symmetric equilibrium) for $(u(w_P + w_{NP}),$ $u(2w_P)) = (1.5, 1.51)$.

B.2 Both spouses at the same workplace

We now consider the case where both spouses compete for promotion in the same firm. That is, assume that households 1 and 2 work in the same firm. We denote by e_{11} and e_{12} the two efforts of the two spouses of family 1 and by e_{21} and e_{22} the corresponding efforts of family 2. We use the same indices for the random terms.

In this setup, a household will have either no or one promoted spouse, associated with household consumption utilities $u(2w_{NP})$ and $u(w_P + w_{NP})$, respectively. Two promotions per household are not possible: As in the base model and in subsection B.1, there is one promotion per firm. For each household, there are only three possible outcomes: either member 1 wins the promotion, or member 2 wins the promotion, or neither wins the promotion.

Household 1 wins if either member 1 or member 2 wins the tournament, that is, if

$$\begin{aligned}\text{either } & e_{11}+\epsilon_{11}>\max\left\{e_{12}+\epsilon_{12},e_{21}+\epsilon_{21},e_{22}+\epsilon_{22}\right\}\\ \text{or } & e_{12}+\epsilon_{12}>\max\left\{e_{11}+\epsilon_{11},e_{21}+\epsilon_{21},e_{22}+\epsilon_{22}\right\},\end{aligned}$$

which is equivalent to

$$\begin{aligned}\text{either } & \left(\epsilon_{12}<\epsilon_{11}+e_{11}-e_{12} \text{ AND } \epsilon_{21}<\epsilon_{11}+e_{11}-e_{21} \text{ AND } \epsilon_{22}<\epsilon_{11}+e_{11}-e_{22}\right)\\ \text{or } & \left(\epsilon_{11}<\epsilon_{12}+e_{12}-e_{11} \text{ AND } \epsilon_{21}<\epsilon_{12}+e_{12}-e_{21} \text{ AND } \epsilon_{22}<\epsilon_{12}+e_{12}-e_{22}\right).\end{aligned}$$

The winning probability of family 1 is thus given by

$$\begin{aligned}&\int F\left(x+e_{11}-e_{12}\right)F\left(x+e_{11}-e_{21}\right)F\left(x+e_{11}-e_{22}\right)f\left(x\right)dx\\ &+\int F\left(x+e_{12}-e_{11}\right)F\left(x+e_{12}-e_{21}\right)F\left(x+e_{12}-e_{22}\right)f\left(x\right)dx.\end{aligned}$$

The expected payoff for family 1 can be stated as

$$\begin{aligned}&\int F\left(x+e_{11}-e_{12}\right)F\left(x+e_{11}-e_{21}\right)F\left(x+e_{11}-e_{22}\right)f\left(x\right)dx\Delta u_{NP}\\ &+\int F\left(x+e_{12}-e_{11}\right)F\left(x+e_{12}-e_{21}\right)F\left(x+e_{12}-e_{22}\right)f\left(x\right)dx\Delta u_{NP}\\ &+u\left(2w_{NP}\right)-c\left(e_{11}\right)-c\left(e_{12}\right).\end{aligned} \tag{B.11}$$

Likewise, the expected payoff for family 2 is

$$\begin{aligned}&\int F\left(x+e_{21}-e_{22}\right)F\left(x+e_{21}-e_{11}\right)F\left(x+e_{21}-e_{12}\right)f\left(x\right)dx\Delta u_{NP}\\ &+\int F\left(x+e_{22}-e_{21}\right)F\left(x+e_{22}-e_{11}\right)F\left(x+e_{22}-e_{12}\right)f\left(x\right)dx\Delta u_{NP}\\ &+u\left(2w_{NP}\right)-c\left(e_{21}\right)-c\left(e_{22}\right).\end{aligned}$$

We obtain the following four first-order conditions:

$$
\begin{aligned}
&\int \frac{\partial \left(F\left(x+e_{11}-e_{12}\right)F\left(x+e_{11}-e_{21}\right)F\left(x+e_{11}-e_{22}\right)\right)}{\partial e_{11}} f\left(x\right)dx\Delta u_{NP} \\
&\quad -\int f\left(x+e_{12}-e_{11}\right)F\left(x+e_{12}-e_{21}\right)F\left(x+e_{12}-e_{22}\right)f\left(x\right)dx\Delta u_{NP} \\
&= c'\left(e_{11}\right), \\
&-\int f\left(x+e_{11}-e_{12}\right)F\left(x+e_{11}-e_{21}\right)F\left(x+e_{11}-e_{22}\right)f\left(x\right)dx\Delta u_{NP} \\
&\quad +\int \frac{\partial \left(F\left(x+e_{12}-e_{11}\right)F\left(x+e_{12}-e_{21}\right)F\left(x+e_{12}-e_{22}\right)\right)}{\partial e_{12}} f\left(x\right)dx\Delta u_{NP} \\
&= c'\left(e_{12}\right),
\end{aligned}
$$

$$
\begin{aligned}
&\int \frac{\partial \left(F\left(x+e_{21}-e_{22}\right)F\left(x+e_{21}-e_{11}\right)F\left(x+e_{21}-e_{12}\right)\right)}{\partial e_{21}} f\left(x\right)dx\Delta u_{NP} \\
&\quad -\int f\left(x+e_{22}-e_{21}\right)F\left(x+e_{22}-e_{11}\right)F\left(x+e_{22}-e_{12}\right)f\left(x\right)dx\Delta u_{NP} \\
&= c'\left(e_{21}\right),
\end{aligned}
$$

and

$$
\begin{aligned}
&-\int f\left(x+e_{21}-e_{22}\right)F\left(x+e_{21}-e_{11}\right)F\left(x+e_{21}-e_{12}\right)f\left(x\right)dx\Delta u_{NP} \\
&\quad +\int \frac{\partial \left(F\left(x+e_{22}-e_{21}\right)F\left(x+e_{22}-e_{11}\right)F\left(x+e_{22}-e_{12}\right)\right)}{\partial e_{22}} f\left(x\right)dx\Delta u_{NP} \\
&= c'\left(e_{22}\right).
\end{aligned}
$$

Notice that

$$
\begin{aligned}
&\frac{\partial \left(F\left(x+e_{11}-e_{12}\right)F\left(x+e_{11}-e_{21}\right)F\left(x+e_{11}-e_{22}\right)\right)}{\partial e_{11}} \\
=&f\left(x+e_{11}-e_{12}\right)F\left(x+e_{11}-e_{21}\right)F\left(x+e_{11}-e_{22}\right) \\
&+F\left(x+e_{11}-e_{12}\right)f\left(x+e_{11}-e_{21}\right)F\left(x+e_{11}-e_{22}\right) \\
&+F\left(x+e_{11}-e_{12}\right)F\left(x+e_{11}-e_{21}\right)f\left(x+e_{11}-e_{22}\right),
\end{aligned}
$$

which means that the first of the first-order conditions becomes

$$\begin{aligned}
&\int f\left(x+e_{11}-e_{12}\right) F\left(x+e_{11}-e_{21}\right) F\left(x+e_{11}-e_{22}\right) f(x)\, dx \Delta u_{NP} \\
&+\int F\left(x+e_{11}-e_{12}\right) f\left(x+e_{11}-e_{21}\right) F\left(x+e_{11}-e_{22}\right) f(x)\, dx \Delta u_{NP} \\
&+\int F\left(x+e_{11}-e_{12}\right) F\left(x+e_{11}-e_{21}\right) f\left(x+e_{11}-e_{22}\right) f(x)\, dx \Delta u_{NP} \\
&-\int f\left(x+e_{12}-e_{11}\right) F\left(x+e_{12}-e_{21}\right) F\left(x+e_{12}-e_{22}\right) f(x)\, dx \Delta u_{NP} \\
=&c'\left(e_{11}\right).
\end{aligned}$$

Now consider the 'specialization' candidate $e_{11} = e_{21} = e_H > e_L = e_{12} = e_{22}$, where $\Delta = e_H - e_L$. The f.o.c. becomes

$$\begin{aligned}
&\int f(x+\Delta) F(x) F(x+\Delta) f(x)\, dx \Delta u_{NP} \\
&+\int F(x+\Delta) f(x) F(x+\Delta) f(x)\, dx \Delta u_{NP} \\
&+\int F(x+\Delta) F(x) f(x+\Delta) f(x)\, dx \Delta u_{NP} \\
&-\int f(x-\Delta) F(x-\Delta) F(x) f(x)\, dx \Delta u_{NP}=2 d e_H .
\end{aligned} \tag{B.12}$$

The second f.o.c. can similarly be rewritten as

$$\begin{aligned}
&-\int f(x+\Delta) F(x) F(x+\Delta) f(x)\, dx \Delta u_{NP} \\
&+\int f(x-\Delta) F(x-\Delta) F(x) f(x)\, dx \Delta u_{NP} \\
&+\int F(x-\Delta) f(x-\Delta) F(x) f(x)\, dx \Delta u_{NP} \\
&+\int F(x-\Delta) F(x-\Delta) f(x) f(x)\, dx \Delta u_{NP}=2 d e_L .
\end{aligned} \tag{B.13}$$

Subtracting (B.13) from (B.12) yields

$$
\begin{aligned}
&2\int f(x+\Delta)\,F(x)\,F(x+\Delta)\,f(x)\,dx\Delta u_{NP}\\
&+\int F(x+\Delta)\,f(x)\,F(x+\Delta)\,f(x)\,dx\Delta u_{NP}\\
&+\int F(x+\Delta)\,F(x)\,f(x+\Delta)\,f(x)\,dx\Delta u_{NP}\\
&-\int F(x-\Delta)\,f(x-\Delta)\,F(x)\,f(x)\,dx\Delta u_{NP}\\
&-\int F(x-\Delta)\,F(x-\Delta)\,f(x)\,f(x)\,dx\Delta u_{NP}\\
&-2\int f(x-\Delta)\,F(x-\Delta)\,F(x)\,f(x)\,dx\Delta u_{NP}=2d\Delta.
\end{aligned}
$$

This can be simplified to

$$
\begin{aligned}
\Bigg(&\int \left(F(x+\Delta)^2-F(x-\Delta)^2\right) f(x)^2dx\\
&+3\int \left(F(x+\Delta)f(x+\Delta)-F(x-\Delta)f(x-\Delta)\right)F(x)f(x)dx\Bigg)\Delta u_{NP}=2d\Delta.
\end{aligned}
\tag{B.14}
$$

Again, we use this equation that only depends on Δ to numerically identify asymmetric equilibrium candidates with $\Delta > 0$. Reinserting Δ-candidates into (B.12) and (B.13), we get candidates for e_H and e_L. Replacing e_{21} and e_{22} in (B.11) with e_H and e_L, and simultaneously maximizing over e_{11} and e_{12}, we can numerically confirm candidates.

For example, assuming the Reflected Exponential distribution (A.6) with scale parameter $\lambda = 2$ as well as $u(2w_{NP}) = 2$ and cost function $c(e) = e^2$, a symmetric equilibrium (but no asymmetric equ.) is obtained for $u(w_P + w_{NP}) = 2.2$ and an asymmetric 'specialization' equilibrium (but no symmetric equ.) for $u(w_P + w_{NP}) = 2.8$.

B.3 Convex combination of joint and individual utility maximization

In this extension, we assume that each spouse independently maximizes a convex combination of household utility and individual utility. Denote by α the weight of joint utility maximization (the basic model) and by $1-\alpha$ the weight of individual maximization.

Denote a spouse's individual payoff from being promoted by $\tilde{u}(w_P)$ and the individual utility from not being promoted by $\tilde{u}(w_{NP})$.

Note: For spouse $1A$'s effort choice it does not matter whether we use $\alpha c(e_{1B})$ or $c(e_{1B})$ as the other spouse's cost term in $1A$'s payoff function, as this term vanishes in the first-order condition with respect to e_{1A}.

We write down the utility of spouse $1A$, i.e., the person in household 1 who chooses effort

e_{1A}:

$$\int F(x+e_{1A}-e_{2A})f(x)dx \cdot \int F(x+e_{1B}-e_{2B})f(x)dx \cdot [\alpha u(2w_P)+(1-\alpha)\tilde{u}(w_P)]$$
$$+\int F(x+e_{1A}-e_{2A})f(x)dx \cdot \left(1-\int F(x+e_{1B}-e_{2B})f(x)dx\right) \cdot [\alpha u(w_P+w_{NP})+(1-\alpha)\tilde{u}(w_P)]$$
$$+\left(1-\int F(x+e_{1A}-e_{2A})f(x)dx\right) \cdot \int F(x+e_{1B}-e_{2B})f(x)dx \cdot [\alpha u(w_P+w_{NP})+(1-\alpha)\tilde{u}(w_{NP})]$$
$$+\left(1-\int F(x+e_{1A}-e_{2A})f(x)dx\right) \cdot \left(1-\int F(x+e_{1B}-e_{2B})f(x)dx\right)$$
$$\cdot [\alpha u(2w_{NP})+(1-\alpha)\tilde{u}(w_{NP})]-c(e_{1A})-\alpha c(e_{1B}). \tag{B.15}$$

For person $1B$, the utility is similar (we only switch $\tilde{u}(w_P)$ and $\tilde{u}(w_{NP})$ in lines 2 and 3 below, and adjust the effort cost):

$$\int F(x+e_{1A}-e_{2A})f(x)dx \cdot \int F(x+e_{1B}-e_{2B})f(x)dx \cdot [\alpha u(2w_P)+(1-\alpha)\tilde{u}(w_P)]$$
$$+\int F(x+e_{1A}-e_{2A})f(x)dx \cdot \left(1-\int F(x+e_{1B}-e_{2B})f(x)dx\right) \cdot [\alpha u(w_P+w_{NP})+(1-\alpha)\tilde{u}(w_{NP})]$$
$$+\left(1-\int F(x+e_{1A}-e_{2A})f(x)dx\right) \cdot \int F(x+e_{1B}-e_{2B})f(x)dx \cdot [\alpha u(w_P+w_{NP})+(1-\alpha)\tilde{u}(w_P)]$$
$$+\left(1-\int F(x+e_{1A}-e_{2A})f(x)dx\right) \cdot \left(1-\int F(x+e_{1B}-e_{2B})f(x)dx\right)$$
$$\cdot [\alpha u(2w_{NP})+(1-\alpha)\tilde{u}(w_{NP})]-\alpha c(e_{1A})-c(e_{1B}). \tag{B.16}$$

The first-order condition for person $1A$ (resp., $1B$) with respect to e_{1A} (resp., e_{1B}) is

$$\int f(x+e_{1A}-e_{2A})f(x)dx \cdot \int F(x+e_{1B}-e_{2B})f(x)dx \cdot [\alpha u(2w_P)+(1-\alpha)\tilde{u}(w_P)]$$
$$+\int f(x+e_{1A}-e_{2A})f(x)dx \cdot \left(1-\int F(x+e_{1B}-e_{2B})f(x)dx\right) [\alpha u(w_P+w_{NP})+(1-\alpha)\tilde{u}(w_P)]$$
$$-\int f(x+e_{1A}-e_{2A})f(x)dx \cdot \int F(x+e_{1B}-e_{2B})f(x)dx \cdot [\alpha u(w_P+w_{NP})+(1-\alpha)\tilde{u}(w_{NP})]$$
$$-\int f(x+e_{1A}-e_{2A})f(x)dx \cdot \left(1-\int F(x+e_{1B}-e_{2B})f(x)dx\right) \cdot [\alpha u(2w_{NP})+(1-\alpha)\tilde{u}(w_{NP})]$$
$$=c'(e_{1A}) \tag{B.17}$$

and

$$
\begin{aligned}
&\int F(x+e_{1A}-e_{2A})f(x)dx \cdot \int f(x+e_{1B}-e_{2B})f(x)dx \cdot [\alpha u(2w_P)+(1-\alpha)\tilde{u}(w_P)] \\
&-\int F(x+e_{1A}-e_{2A})f(x)dx \cdot \int f(x+e_{1B}-e_{2B})f(x)dx \cdot [\alpha u(w_P+w_{NP})+(1-\alpha)\tilde{u}(w_{NP})] \\
&+\left(1-\int F(x+e_{1A}-e_{2A})f(x)dx\right) \cdot \int f(x+e_{1B}-e_{2B})f(x)dx \cdot [\alpha u(w_P+w_{NP})+(1-\alpha)\tilde{u}(w_P)] \\
&-\left(1-\int F(x+e_{1A}-e_{2A})f(x)dx\right) \int f(x+e_{1B}-e_{2B})f(x)dx \cdot [\alpha u(2w_{NP})+(1-\alpha)\tilde{u}(w_{NP})] \\
&= c'(e_{1B}).
\end{aligned} \tag{B.18}
$$

Denoting $\Delta\tilde{u} := \tilde{u}(w_P) - \tilde{u}(w_{NP})$, these can be simplified to

$$
\begin{aligned}
&\int f(x+e_{1A}-e_{2A})f(x)dx \left[\alpha \left(\int F(x+e_{1B}-e_{2B})f(x)dx\Delta u_P \right.\right. \\
&\left.\left.+\left(1-\int F(x+e_{1B}-e_{2B})f(x)dx\right)\Delta u_{NP}\right)+(1-\alpha)\Delta\tilde{u}\right] = c'(e_{1A})
\end{aligned} \tag{B.19}
$$

and

$$
\begin{aligned}
&\int f(x+e_{1B}-e_{2B})f(x)dx \left[\alpha \left(\int F(x+e_{1A}-e_{2A})f(x)dx\Delta u_P \right.\right. \\
&\left.\left.+\left(1-\int F(x+e_{1A}-e_{2A})f(x)dx\right)\Delta u_{NP}\right)+(1-\alpha)\Delta\tilde{u}\right] = c'(e_{1B})
\end{aligned} \tag{B.20}
$$

Now consider the asymmetric 'specialization' candidate $((e_{1A}, e_{1B}), (e_{2A}, e_{2B})) = ((e_H, e_L), (e_L, e_H))$. Denoting $\Delta := e_H - e_L$, such that $e_{1A} - e_{2A} = \Delta$ and $e_{1B} - e_{2B} = -\Delta$, and inserting into (B.19) and (B.20), the two first-order conditions can be written as

$$
\begin{aligned}
&\int f(x+\Delta)f(x)dx \left[\alpha \left(\int F(x-\Delta)f(x)dx\Delta u_P \right.\right. \\
&\left.\left.+\left(1-\int F(x-\Delta)f(x)dx\right)\Delta u_{NP}\right)+(1-\alpha)\Delta\tilde{u}\right] = c'(e_H)
\end{aligned} \tag{B.21}
$$

and

$$
\begin{aligned}
&\int f(x-\Delta)f(x)dx \left[\alpha \left(\int F(x+\Delta)f(x)dx\Delta u_P \right.\right. \\
&\left.\left.+\left(1-\int F(x+\Delta)f(x)dx\right)\Delta u_{NP}\right)+(1-\alpha)\Delta\tilde{u}\right] = c'(e_L)
\end{aligned} \tag{B.22}
$$

Recalling $c'(e) = 2de$, subtracting (B.22) from (B.21), we get

$$\begin{aligned}
&\int f(x+\Delta)f(x)dx \left[\alpha \left(\int F(x-\Delta)f(x)dx\Delta u_P \right.\right.\\
&\left.\left. + \left(1 - \int F(x-\Delta)f(x)dx\right)\Delta u_{NP}\right) + (1-\alpha)\Delta\tilde{u}\right]\\
&- \left(\int f(x-\Delta)f(x)dx \left[\alpha \left(\int F(x+\Delta)f(x)dx\Delta u_P \right.\right.\right.\\
&\left.\left.\left. + \left(1 - \int F(x+\Delta)f(x)dx\right)\Delta u_{NP}\right) + (1-\alpha)\Delta\tilde{u}\right]\right) = 2d\Delta,
\end{aligned} \tag{B.23}$$

i.e., a single equation that depends on Δ only. This equation can be used to numerically identify asymmetric equilibrium candidates. These Δ-candidates can then be reinserted into (B.21) and (B.22) to get candidates for e_H and e_L.

For example, assume that the weight of joint utility is 70%, i.e., $\alpha = 0.7$, (compared to 100% in the base model) whereas individual utility has weight 30% in each spouse's payoff function. Now, assuming the Reflected Exponential distribution (A.6) with scale parameter $\lambda = 2$ and cost function $c(e) = e^2$, a symmetric equilibrium (but no asymmetric equ.) is obtained for $(u(2w_{NP}), u(w_P + w_{NP}), u(2w_P)) = (1, 2, 2.1)$ and $(\tilde{u}(w_{NP}), \tilde{u}(w_P)) = (1, 2)$.

We obtain an asymmetric 'specialization' equilibrium (but no symmetric equ.) for $(u(2w_{NP}), u(w_P + w_{NP}), u(2w_P)) = (1, 2.7, 2.71)$ and $(\tilde{u}(w_{NP}), \tilde{u}(w_P)) = (1, 2.7)$.

B.4 Household production effort requirement

Assuming the Reflected Exponential distribution (A.6) with scale parameter $\lambda = 2$ as well as $u(2w_{NP}) = 1$ and cost function $c(e) = e^2$, an asymmetric 'specialization' equilibrium (but no symmetric equ.) arises for $(u(w_P + w_{NP}), u(2w_P)) = (3, 3.1)$. The equilibrium efforts in this example are $(e_H, e_L) = (0.448735, 0.232182)$.

We now introduce a fixed household-production effort of $\bar{e}_{HP} = 0.11$ which is roughly 50% of e_L, which seems to be a non-trivial amount of household production for this example. We have confirmed that the modified game has an asymmetric 'specialization' equilibrium with promotion efforts $(\tilde{e}_{1A}, \tilde{e}_{1B}) = (\tilde{e}_H, \tilde{e}_L) = (0.380764, 0.0332649) = (\tilde{e}_{2B}, \tilde{e}_{2A})$ in which spouses $1B$ and $2A$ contribute the household production. In this example, the effort reductions relative to the base game are $e_H - \tilde{e}_H = 0.067971$ and $e_L - \tilde{e}_L = 0.198917$. In total, the reduction is larger than the amount of household production, which is intuitive given the convex effort cost functions. In percentages, e_H is reduced by 15 % while e_L is reduced by roughly 86%.